HW Puzzles: Week 3 (Monsky's Theorem)

September 24, 2026

Puzzle 1. *Show that a regular $2n$-gon cannot be partitioned into an odd number of triangles of equal area.*

Puzzle 2. *Consider the cube $[0, n]^d \subset \mathbb{R}^d$ partitioned into n^d unit cubes in the most obvious way. Label each vertex $(x_1, \ldots, x_d)$ of this partition by a binary vector $(b_1, \ldots, b_d)$ in $\{0, 1\}^d$ in the following manner:*

1. *If $x_i = 0$, then $b_i = 0$.*
2. *If $x_i = n$, then $b_i = 1$.*
3. *If there are two neighboring vertices, their labels differ in AT MOST one coordinate.*

Show that there must be a unit cube whose vertices carry all possible 2^d labels.

Puzzle 3. *Show that if the unit cube in $\mathbb{R}^3$ is partitioned into n tetrahedra of equal volume, then n must be even,*

Puzzle 4. *Find a sequence of nonzero rationals that converges to 0 in the p-adic norm for every prime p and also in $\mathbb{R}$, or show there is no such sequence.*

Puzzle 5. *Let n be odd and assume that the unit square $[0, 1]^2$ is partitioned into n triangles. By Monsky's theorem we know that these triangles cannot have equal areas. Denote by $M = M(n)$ the minimum (over all possible triangulations) of the difference between the largest area of a triangle and the smallest area of a triangle in the partition. It is not hard to show that $M(n) = O(\frac{1}{n})$. Can you show better upper bounds? More interestingly, can you show any lower bound for $M(n)$ in terms of n?*