

HW Puzzles: Week 2 (Distances)

September 17, 2026

Puzzle 1. *Given n red points, n blue points, and n green points in the plane. Is it true that the sum of the areas of the rainbow (with vertices of three colors) triangles is always greater than or equal to the sum of the areas of the monochromatic (with same color vertices) triangles?*

Puzzle 2. *Given n points in the unit ball (any dimension, or even only in the plane), what is the maximum possible sum of distances between pairs of points?*

Puzzle 3. *Given n points in the plane, consider all the $\binom{n}{3}$ triangles determined by triples of points. At most how many of these triangles have the maximum area among all triangles?*

Puzzle 4. *n straight line segments are drawn in the plane and they pairwise cross each other. Show that it is not possible to draw different n segments with the same $2n$ endpoints of the original segments such that the sum of their lengths is larger than, or even equal to, the sum of the the lengths of the original n segments.*

Puzzle 5. *n blue points and n red points are on a sphere in $\mathbb{R}^3$. Show that the sum of the monochromatic distances is smaller than or equal to the sum of the bichromatic distances. Here the distances are on the sphere. That is, the distance between two points is the length of the shortest (great) arc between the two points on the sphere.*

Suggestion: try first to solve the same problem on the circle.