HW Puzzles: Week 1

February 19, 2026

Puzzle 1. *Show that if we can tile a $a \times b$ grid of squares by horizontal and vertical parts of sizes $1 \times k$ and $k \times 1$, then a or b must be an integer multiple of k.*

Puzzle 2. *We are given a grid of $a \times b$ squares of, colored black and white like a chessboard. Show that if ab is an even number, then no matter how we remove one white square and one black square, the remaining $ab-2$ squares can be tiled by domino pieces.*

Puzzle 3. *Assume a 3-dimensional box B is partitioned into smaller boxes $B_1, \ldots, B_n$. Each of the smaller boxes has one edge that is an integer multiple of k (k need not be an integer). Show that also the big box B has an edge that is an integer multiple of k.*

Puzzle 4. *We are given an odd number of translates of a fixed triangle T in the plane. Prove that the area of those points that belong to an odd number of these triangles is at least one half of the area of T.*

(Suggestion: by applying a linear transformation, we may assume that T is the triangle with vertices $(1,0), (0,1)$, and $(0,0)$. Find an appropriate coloring).

Puzzle 5. *Let N be fixed and let p be a large enough prime (how large should be p?). Show that among $2N-1$ nonzero numbers in $\mathbb{F}_p$ one can always choose N of numbers with all nonempty subsets having nonzero sums.*