Discrete Optimization 2024 (EPFL): Problem set of week 11

May 8, 2024

1. a) What is the minimum vertex cover for the complete graph on n vertices (the graph on n vertices where every two vertices are connected by an edge)?

   b) How large can be the minimum vertex cover for a tree with n vertices?

2. We saw that in bipartite graph the maximum size of a matching is equal to the minimum size of a vertex cover. In general graphs the minimum vertex cover is greater than or equal to the maximum size of a matching. Show that it is always true that the minimum vertex cover is at most twice the size of the maximum matching in a graph. For every n find a graph with maximum matching equal to n and minimum vertex cover equal to $2n$.

3. Write a linear program that finds a minimum set (if there is one) of edges of a given bipartite graph G that together contain all the vertices of G (as usual we assume G has n vertices and m edges).

4. Write a linear program that finds a set (if there is one) of n edges in a bipartite graph G (on n vertices and m edges) that together form a union of disjoint cycles.