Linear Programming 2024 (EPFL): Problem set of week 10

May 3, 2024

1. We are given m subsets $S_1, \ldots, S_m$ of $\{1, \ldots, n\}$.

   a) Find an integer linear program that computes the maximum number of pairwise disjoint sets among $S_1, \ldots, Sm$.

   b) Find an integer linear program that computes the minimum cardinality of a subset B of $\{1, \ldots, n\}$ such that $B \cap S_i$ is not empty for every i.

2. Let A be a matrix where column of A contains only 0's except for one coordinate that is equal to 1 and another coordinate that is equal to -1. Show that A is totally unimodular.

3. Give an example for a linear program with no maximum (in other words, unbounded linear program) such that the corresponding integer program is not unbounded.

4. Let A be a matrix where every row looks like $(0, \ldots, 0, 1, \ldots, 1, 0, \ldots, 0)$, or $(1, \ldots, 1, 0, \ldots, 0)$, or $(0, \ldots, 0, 1, \ldots, 1)$, or $(1, \ldots, 1)$. That is, all the 1's appear in one interval. Show that every $k \times k$ submatrix of A has determinant $0, 1$, or -1 (in other words, A is totally unimodular).

5. Let K be the cone generated by n linearly independent vectors in $\mathbb{R}^n$. Show that C must contain infinitely many integer points.