

Discrete Optimization 2024 (EPFL): Problem set of week 1

February 22, 2024

Some of the exercises might be more challenging than usual.

1. Let $v_1, \dots, v_{n+1} \in \mathbb{R}^n$ be such that the angles between every two are equal. Find this angle if it is known that it is different from 0.
2. Let Δ be a triangle in \mathbb{R}^n such that all the coordinates of its vertices are integers. Show that the area of Δ is at least $\frac{1}{2}$.
3. Find all the vectors in \mathbb{R}^3 that are perpendicular to $(1, 1, 1)$ and create an angle of 60 degrees with $(1, 2, 3)$
4. $(1, 2, 3)$, $(2, -4, 5)$, and $(-2, 0, 9)$ are three vertices of a parallelogram in \mathbb{R}^3 . What are the possibilities for the fourth vertex?
5. Show that it is impossible to find $n + 1$ nonzero vectors in \mathbb{R}^n every two of which are perpendicular.