Discrete Optimization 2024 (EPFL): Problem set of week 8

May 2, 2024

Reminder: The min-max theorem for zero-sum games with mixed strategies says that for every $m \times n$ matrix A we have

$$\min_y \max_x yAx = \max_x \min_y yAx,$$

where the minimum is over all $y = (y_1, \dots, y_m) \geq 0$ such that $\sum y_i = 1$. The maximum is over all $x = (x_1, \dots, x_n) \geq 0$ such that $\sum x_i = 1$.

1. Let A be an $m \times n$ matrix. Assume that there is an entry in A that is the minimum in its column and the maximum in its row. Prove that this entry is the value of the zero-sum game with for two players with mixed strategies.

2. Find $\max_x \min_y yAx$ for the matrix

$$A = \begin{pmatrix} 5 & 1 \\ 3 & 4 \end{pmatrix}$$

   What is the (mixed) strategy for the column player to guarantee the maximum possible result?

3. Find the min-max value for the diagonal matrix with $\lambda_1, \dots, \lambda_n$ on the main diagonal.

4. Show that in a zero-sum game with a matrix A with mixed strategies the following is true: If one player knows the mixed strategy of the other player, then the best response (strategy) for him is a pure strategy. That is, the best response is choosing just one row or column.

5. Find a hyperplane separating the point $v = (4, 3, 6)$ from the ball $\{(x, y, z) \mid (x-1)^2 + (y-2)^2 + (z-3)^2 \leq 16\}$.