

Graph Theory 2023 (EPFL): Problem set of week 14

December 21, 2023

1. Let G be a graph on n vertices. Show that the edges of G can be partitioned into at most $n/2$ tours. The tours can be closed or not but no two tours can share an edge.
2. Let M be a planar map representing a crossing free drawing of a planar bipartite graph. Prove that there is a closed curve in the plane that crosses every edge of M precisely once.
3. Let G be a graph. The edge-graph of G that we denote by G' is a graph whose vertices are the edges of G . Two vertices of G' are connected by an edge if the corresponding edges in G share a vertex.
Show that if G has Euler cycle, then G' has a Hamilton cycle and also an Euler cycle.
4. Let G be a graph on n vertices with $(n-1)(n-2)/2 + 2$ edges. Show that G has a Hamilton cycle. Give an example for a graph with n vertices and $(n-1)(n-2)/2 + 1$ edges that does not have a Hamilton cycle.
5. n tennis players play $\binom{n}{2}$ games with one another so that every two play once. Prove that it is always possible to arrange the people in a row such that every person (except for the leftmost) won the person standing to its left.