Graph Theory 2023 (EPFL): Problem set of week 13

December 14, 2023

1. Show that for any k there is $n(k)$ such that in any set of $n > n(k)$ points in $\mathbb{R}^3$ either there are k points on the same 2-dimensional plane, or there are k points no 4 of them lie on a common plane.

2. Show that for every k there is $n(k)$ such that if $n > n(k)$ and we color the set of all rational numbers $\frac{a}{b}$ such that $1 \leq a < b \leq n$ by k colors, then one can find a monochromatic triple of such rational numbers x, y, z such that $xy = z$.

3. Let G be an infinite graph. That is, a graph on a set of vertices that is infinite. Prove that if G is connected (there is a path between any two vertices), then either there is a vertex of infinite degree in G, or there is an infinite path in G (could be that both exist).

4. Let k be fixed. Prove that for any coloring of the two dimensional integer grid points (these are points of the form (a, b), where both a and b integers) with k one can find integers $x_1 < \ldots < x_{100}$ and $y_1 < \ldots < y_{100}$ such that all the points (x_i, y_j) have the same color.