

Graph Theory 2023 (EPFL): Problem set of week 12

December 7, 2023

1. Prove that in any coloring of the edges of K_6 with two colors there are at least **two** monochromatic triangles.
2. Show that for every r there exists $n(r)$ such that for any set of $n > n(r)$ points in the plane there exists a coloring of the $\binom{n}{2}$ segments connecting the n points by two colors there is a monochromatic path of length r going from left to right (in the sense that it is advancing) and all of whose edges are with positive slopes, or all of whose edges are with negative slopes.
3. We are given a set of n segments in the plane. It is known that no 100 segments may be pairwise intersecting. Prove that if n is large enough, then one can find among the segments 200 segments that are pairwise disjoint.
4. Show that for every r there is $n(r)$ such that for every $n > n(r)$ and any coloring of the edges of the complete bipartite graph $K_{n,n}$ in two colors, there is a monochromatic complete bipartite subgraph $K_{r,r}$.
5. Prove that if n is large enough, then no matter how we place n points in the plane, one can always find 1000 pairwise crossing segments connecting pairs of our points.