

# Graph Theory 2023 (EPFL): Problem set of week 9

November 15, 2023

1. Given  $n$  points in the plane such that no three of them are on a line. Show that one can find at least  $cn^4$  (for some absolute constant  $c$ ) quadruples of points that form the set of vertices of a convex quadrilateral.
2. Find geometric graphs with  $n$  vertices and  $e$  edges and only  $ce^3/n^2$  crossings for every  $e$  and  $n$ .  
Hint: consider a set of  $n$  points on a circle. Then take the "shortest" possible chords.
3.  $G$  is a graph with  $n$  vertices that can be drawn in the plane in such a way that every edge is crossed by at most one other edge. Show that the number of edges in  $G$  is at most  $10n$ .
4. Let  $G$  be a graph with  $n$  vertices and  $e$  edges. Prove that  $G$  has at least  $ce^3/n^2$  distinct paths of three edges.