

# Graph Theory 2023 (EPFL): Problem set of week 8

November 9, 2023

1. Let  $P$  be a regular  $n$ -gon in the plane. Prove that it is not possible to draw more than  $n - 3$  diagonals of  $P$  without two of them crossing each other.
2. Let  $G$  be a planar graph. Show that  $G$  is bi-partite if and only if there is a drawing of  $G$  in the plane where all the bounded faces have are of even size (that is, quadrangles, hexagons, etc.).
3. Let  $G$  be a planar graph on  $n$  vertices that can be drawn as a planar map in the plane such that every face in this map has size at least  $k$ . Show that the number of edges in  $G$  is at most  $\frac{k}{k-2}n - \frac{2k}{k-2}$ .
4. What are all the essentially different planar maps for which it is known that all the vertices have the same degree and all the faces have the same size?
5. Let  $P$  be a set of  $n$  points in the plane such that no three of them are on one line. for every two points draw the line segment connecting them. Show that if  $n \geq 7$  (in fact even for  $n \geq 5$ ), then there is always a point in the plane that belongs to precisely two of the segments.