Graph Theory 2023 (EPFL): Problem set of week 3

October 4, 2023

1. Show that if $d_1, \dots, d_n$ are n natural numbers such that $\sum_{i=1}^{n} d_i = 2n-2$, then there is a tree T whose set of degrees is precisely $d_1, \dots, d_n$.

2. a) Let T be a tree and let e be an edge not in T. Show that if we add e to T we get a graph with precisely one cycle.

   b) Show that if T is a tree and we add to T k red egdes that are not in T, then the resulting graph has at most $2^k - 1$ distinct cycles.

   Hint: show that it is not possible that two different cycles use the same set of red edges.

3. a) It is known that T is a tree with 10 vertices of degree 10 and all other vertices are leaves. How many vertices does T have?

   b) How many different trees on n labeled vertices are there such that the degree of each vertex is either 3 or 1?

4. Show that when n is even, then the complete graph K_n (that has $(n-1)n/2$ edges) is a union of $n/2$ trees on the same set of vertices. In other words: show that the set of edges of the complete graph K_n can be partitioned into $n/2$ sets of $n-1$ edges such that each set of $n-1$ edges forms a tree on the set of vertices of K_n.

   Hint: there is more than one way to do it. One way is induction.