Graph Theory 2023 (EPFL): Problem set of week 5

October 19, 2023

1. Show that if G does not contain a cycle of even length, then no two cycles in G may share an edge.

2. Let G be a graph on n vertices. Consider the matrix E as we defined in class where the rows correspond to the vertices and the columns to the edges. We make a small change and each column has two 1's at the entries that correspond to the vertices of the edge (rather than $+1$ and -1 as we had in class).

   Show that the determinant of n columns of E is non-zero if and only if the corresponding n edges form a subgraph H of G that has precisely one cycle, necessarily of odd length, in each connected component of H.

3. Let G be a graph with no cycle of even length. Show that G has at most $\frac{3}{2}n$ edges.

4. Let n points be given on a circle of radius 1 in the plane. Show that at least $\frac{n^2}{10} - \frac{3n}{2}$ pairs of the given points points are at distance smaller than 1.