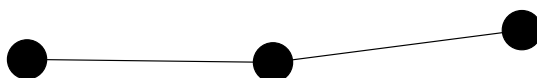


Graph Theory 2023 (EPFL): Problem set of week 2

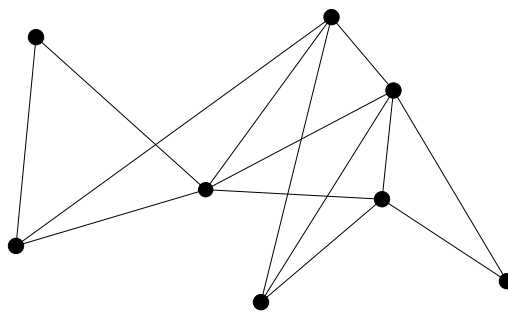
September 28, 2023

1. Let G be the graph that is a cycle of length 4. Find all the eigenvalues and eigenvectors of $A(G)$ and determine the number of walks of length 100 between every two distinct vertices of G .
2. Let G be a path of length 2. That is, G is as in the following figure:



In how many ways one can get from one end of the path to the other in 100 steps? In 2023 steps?

3. Solve the light bulbs puzzle that we saw in class for the following specific graph. We imagine that there is a light bulb in each vertex. Pressing the switch at a vertex changes the condition of the vertex and its neighbors. Find a way to turn ON all light bulbs. At the start they are all OFF.



4. Solve the following puzzle that is a variation of the light bulbs puzzle that we saw in class:

We get a graph G and at every vertex there is a light bulb and a switch. Some of the switches are red and some are blue. If we press a red switch it changes the condition of the light bulbs of the vertex and all its neighbors in G . If we press a blue switch then it changes the condition only of the neighbors of the vertex but not the bulb of the vertex itself. Show that we can always make it happen that precisely all the light bulbs with the red switches are turned ON and the light bulbs with the blue switches are turned OFF. When we start everybody is turned OFF.