Graph Theory 2023 (EPFL): Problem set of week 1

September 20, 2023

1. The complement $\overline{G}$ of a graph G is a graph on the same set of vertices such that any two vertices in $V(G) = V(\overline{G})$ are connected by an edge in G if and only if they are not connected by an edge in $\overline{G}$.

   Show that either G, or $\overline{G}$ (or both) must be a *connected* graph. A connected graph is a graph where every two vertices can be connected by a path of edges.

2. Two graphs G and H are called isomorphic if there is a one to one and onto map f from $V(G)$ to $V(H)$ such that $x, y \in V(G)$ are connected by an edge in G if and only if $f(x)$ and $f(y)$ are connected by an edge in H.

   For every $n \geq 6$ give an example of two graphs on n vertices that are not isomorphic but have the same set of degrees of vertices.

3. a) Give two different examples for graphs G that are isomorphic to their complement $\overline{G}$.

   b) Can you find a graph G on 2023 vertices that is isomorphic to its complement $\overline{G}$?

4. There is a group of n kids. Some pairs of kids are friends, some are not. Show that one can always divide the kids into two groups such that every kid has at least as many friends in the other group than in his/her own group.

   Conclude from here (what we have already seen in class) that every graph G contains a bipartite subgraph H with at least half of the edges of G.