

# Discrete Optimization 2023 (EPFL): Problem set of week 12

May 26, 2023

1. Consider the polyhedron  $P$  defined by  $Ax \leq b$  for

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 2 & 0 & 1 \\ -1 & 1 & -2 \\ 3 & 1 & -1 \\ 2 & 2 & 1 \end{pmatrix}$$

and  $b = (1, 2, 3, 4, 5)$ .

Find a hyperplane separating  $P$  and  $x = (1, 2, 3)$ .

2. Find the volume of the cross polytope, the polytope with the  $2n$  vertices in  $\mathbb{R}^n$  that are  $\pm e_1, \dots, \pm e_n$ .
3. Let  $P$  be a tetrahedron with  $n + 1$  integer vertices in  $\mathbb{R}^n$ . Assume that  $P$  contains an integer interior point. Prove that the volume of  $P$  is at least  $\frac{1}{(n-1)!}$ .
4. Let  $T$  be a non-singular integer linear transformation. Assume that the entries in the matrix representing  $T$  are at most  $D$  in absolute value. By how much can  $T$  shrink the size of a vector at most? That is, how small can  $\frac{|Tx|}{|x|}$  be?