Discrete Optimization 2023 (EPFL): Problem set of week 10

May 5, 2023

1. Let A be a matrix where every row looks like $(0, \ldots, 0, 1, \ldots, 1, 0, \ldots, 0)$, or $(1, \ldots, 1, 0, \ldots, 0)$, or $(0, \ldots, 0, 1, \ldots, 1)$, or $(1, \ldots, 1)$. That is, all the 1's appear in one interval. Show that every $k \times k$ submatrix of A has determinant $0, 1$, or -1 (in other words, A is totally unimodular).

2. Let A be a matrix where each column of A contains only 0's except for one coordinate that is equal to 1 and another coordinate that is equal to -1. Show that every $k \times k$ submatrix of A has determinant $0, 1$, or -1 (in other words, A is totally unimodular).

3. Let ℓ be a line through the origin O (could be any other integer point as well). Let $\epsilon > 0$ be any positive number and consider the cylinder S that consists of all points at distance at most ϵ from ℓ. Show that S must contain infinitely many integer points.

4. Let K be the cone generated by n linearly independent vectors in $\mathbb{R}^n$. Show that K must contain infinitely many integer points.

5. Give an example for a linear program with no maximum (in other words, unbounded linear program) such that the corresponding integer program is not unbounded.