

Discrete Optimization 2023 (EPFL): Problem set of week 12

May 19, 2023

1. Verify that $\{(x, y) \in \mathbb{R}^2 \mid 5x^2 - 2xy + 2y^2 \leq 1\}$ is an ellipse and find its area.
2. Let B be the unit ball in \mathbb{R}^3 and let H be the hyperplane $H = \{x + 3y + 2z = 0\}$.

Consider the linear transformation

$$T = \begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

What is the (two dimensional) area of $T(B \cap H)$?

3. Let $E \subset \mathbb{R}^3$ be the ellipsoid $E = \{(x, y, z) \mid \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} \leq 1\}$.
Let H be the hyperplane $H = \{(x, y, z) \mid x - 3y + 2z = 0\}$.
Find the area of $E \cap H$.
4. Let $E \subset \mathbb{R}^3$ be the ellipsoid $E = \{(x, y, z) \mid x^2 + \frac{y^2}{4} + \frac{z^2}{9} \leq 1\}$.
Let H^+ be the half-space $H^+ = \{(x, y, z) \mid x + y + z \geq 0\}$.
Find an ellipsoid E' such that $E' \supset E \cap H^+$ and $\text{Vol}(E') \leq \text{Vol}(E)e^{-1/(2(3+1))}$.