Discrete Optimization 2023 (EPFL): Problem set of week 8

April 20, 2023

1. Let K be a cone in $\mathbb{R}^n$. Prove that any hyper-plane H supporting K must pass through the origin O.

2. Prove that $A\overrightarrow{x} = \overrightarrow{b}$ has a solution (we do not require $x \geq 0$ as in Farkas' Lemma) if and only if for every y such that $yA = 0$ we also have $\langle y, b \rangle = 0$.

3. Prove the following Farkas-like Lemma: $Ax < 0, \quad x \geq 0$ has a solution if and only if there is no $y \geq 0, \quad y \neq 0$ such that $yA \geq 0$.

4. Prove the following Farkas-like Lemma: $Ax = 0, \quad x > 0$ has a solution if and only if there is no y such that $yA \geq 0$ and $yA \neq 0$.