Discrete Optimization 2023 (EPFL): Problem set of week 9

April 27, 2023

Reminder: The min-max theorem for zero-sum games with mixed strategies says that for every $m \times n$ matrix A we have

$$\min_y \max_x yAx = \max_x \min_y yAx,$$

where the minimum is over all $y = (y_1, \ldots, y_m) \geq 0$ such that $\sum y_i = 1$. The maximum is over all $x = (x_1, \ldots, x_n) \geq 0$ such that $\sum x_i = 1$.

1. Let A be an $m \times n$ matrix. Assume that there is an entry in A that is the maximum in its column and the minimum in its row. Prove that this entry is the value of the zero-sum game with for two players with mixed strategies.

2. Prove the min-max theorem directly for matrices of the form

$$A = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

3. Find the min-max value for the diagonal matrix with $\lambda_1, \ldots, \lambda_n$ on the main diagonal.

4. Show that in a zero-sum game with a matrix A with mixed strategies the following is true: If one player knows the mixed strategy of the other player, then the best response (strategy) for him is a pure strategy. That is, the best response is choosing just one row or column.