Discrete Optimization 2023 (EPFL): Problem set of week 3

March 1, 2023

1. For any given $\overrightarrow{a} \in \mathbb{R}^n$ and any $b \in \mathbb{R}$ prove algebraically (from the algebraic definitions) that the half-space $\{\overrightarrow{x} \mid \langle x, a \rangle \leq b\}$ is convex.

2. Let Q be the quadrangle in the plane whose vertices are $(4, 3)$, $(3, 4)$, $(2, 3)$ and $(3, 2)$. Find a matrix A and a vector $\overrightarrow{b}$ such that $Q = \{\overrightarrow{v} = (x, y) \mid A\overrightarrow{v} \leq \overrightarrow{b}\}$.

3. Let B be the box in $\mathbb{R}^3$ defined by $B = \{\overrightarrow{v} = (x, y, z) \mid 1 \leq x \leq 5, \quad -2 \leq y \leq 6, \quad 0 \leq z \leq 2\}$. Find a matrix A and a vector $\overrightarrow{b}$ such that $B = \{\overrightarrow{v} = (x, y, z) \mid A\overrightarrow{v} \leq \overrightarrow{b}\}$.

4. Let P be the three dimensional pyramid with vertices $(1, 1, -6)$, $(1, 3, -4)$, $(-1, -2, 5)$, and $(3, 5, 1)$. Find $\overrightarrow{c} \in \mathbb{R}^3$ such that the function $\langle \overrightarrow{c}, (x, y, z) \rangle$ attains its maximum on P precisely at the vertex $(1, 3, -4)$.