

Discrete Optimization 2023 (EPFL): Problem set of week 4

March 8, 2023

1. Let A be the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{pmatrix}.$$

Let $\vec{b} = (1, 1, 1, 1, 1)$ and let $P = \{\vec{v} = (x, y, z) \in \mathbb{R}^3 \mid A\vec{v} \leq \vec{b}\}$. Show that P is a polytope and find all its vertices.

What is the maximum value of $x + 2y + 3z$ on P ?

2. Find a hyperplane separating the ellipsoid $E = \{(x, y, z) \mid 2x^2 + \frac{2y^2}{4} + \frac{2z^2}{9} \leq 2\}$ from the point $p = (1, 2, 3)$.
3. Let A be the $2^n \times n$ matrix whose rows are all the 2^n possible combinations of 1 and -1 . Let $\vec{b} = (1, 1, 1, \dots, 1) \in \mathbb{R}^{2^n}$. Show that $\{\vec{x} \mid A\vec{x} \leq \vec{b}\}$ is a polytope and find all its vertices.
4. Suppose P is a polytope in \mathbb{R}^n . Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^k$ be any linear map. Show that $T(P)$ is a polytope in \mathbb{R}^k .