Discrete Optimization 2023 (EPFL): Problem set of week 2

February 23, 2023

We say that a hyperplane $H = \{x \mid < x, a >= b\}$ separates a point p from a set W, if $< p, a >> b$ while $< x, a >< b$ for every $x \in W$.

1. Find a point that is inside the tetrahedron whose facets are:
   $\{x + y + z = 1\}$, $\{2x - 3y - z = 2\}$, $\{x - 3y + z = 4\}$, and $\{2x - y + 3z = 1\}$.

2. Find a hyperplane separating the point $(1, 2, 3)$ from the unit ball $B = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$ in $\mathbb{R}^3$.

3. Let B be the cube $B = \{(x_1, \ldots, x_6) \mid 0 \leq x_1, \ldots, x_6 \leq 1\}$ in $\mathbb{R}^6$. Find a hyperplane passing through the point $(1, 0, 1, 0, 1, 0)$ that does not contain any other point of B.

4. Let $p_1, \ldots, p_m$ be m points in $\mathbb{R}^n$. Prove that the point $q = \frac{1}{m}(p_1 + \ldots, +p_m)$ cannot be separated by a hyperplane from the points $p_1, \ldots, p_m$.