Hardness of Multi-Stage Stochastic ILPs

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Integer programs play an important role in theory and applications, but are hard to solve in general. In this project, we consider the special case of so-called multi-stage stochastic ILPs, which are recursively composed of 2-stage stochastic ILPs.

The constraint matrix $\mathcal{A}$ of a 2-stage stochastic ILP has the following form:

$$\mathcal{A} = \begin{pmatrix} A_1 & B_1 & 0 & \dots & 0 \\ A_2 & 0 & B_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ A_n & 0 & \dots & 0 & B_n \end{pmatrix},$$

where $A_1, \dots, A_n \in \mathbb{Z}^{t \times r}$ and $B_1, \dots, B_n \in \mathbb{Z}^{t \times s}$ are integer matrices themselves. In the multi-stage stochastic ILP problem, the B_i matrices are (recursive) 2-stage stochastic matrices.

It is known that multi-stage stochastic ILPs can be solved in a running time involving a tower of exponents of height equal to the recursion depth [1]. Several lower and upper bounds for related settings are known, see [1,2,3,4] for an overview.

The goal of this project is to gather the current state-of-the-art lower and upper bounds and to understand the connection between them. Further, a (new) running time lower bound for the multi-stage stochastic ILPs is sought-after.

References

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