Problem 1. In the MaxCut problem we are given an undirected graph on n vertices and our goal is to find a partition of the vertices into two sets that maximizes the number of edges going between the sets. Give an $\mathcal{O}^*(2^{\frac{\omega}{3}n})$ time algorithm for MaxCut.

Problem 2. In the Minimum Consecutive Sums problem we are given n integers $a_1, a_2, \ldots, a_n$ and we need to compute for every $k \in \{1, \ldots, n\}$ the smallest sum of any k consecutive of these integers, i.e.

$$\min_{i \in \{1,\ldots,n-k+1\}} x_i + \cdots + x_{i+k-1}.$$

Prove that if there is a truly subquadratic algorithm for Minimum Consecutive Sums, then there is also a truly subquadratic algorithm for $(\min, +)$-convolution, and vice versa.

Definition. In the Zero 3-Star problem we are given an edge-weighted four-partite graph $G = (V_1 \cup V_2 \cup V_3 \cup V_4, E)$, with $|V_1| = |V_2| = |V_3| = |V_4| = n$ and $E \subseteq V_1 \times (V_2 \cup V_3 \cup V_4)$, and we need to determine if there exist vertices $v_1 \in V_1$, $v_2 \in V_2$, $v_3 \in V_3$, $v_4 \in V_4$ that form a star of total weight 0, i.e. $w(v_1, v_2) + w(v_1, v_3) + w(v_1, v_4) = 0$.

Problem 3. Show that if 3SUM can be solved in truly subquadratic time, then Zero 3-Star can be solved in truly subcubic time.

Problem 4. Show that if Zero 3-Star can be solved in truly subcubic time, then 3SUM can be solved in truly subquadratic time. Hint: Reduce one instance of 3SUM of size n to $\mathcal{O}(k^2)$ instances of size n/k.

*This problem set adds ~~**1 point**~~ **0 points** to the threshold for grade 4.0, and ~~**2 points**~~ **1 point** for 6.0. In the initial version of this problem set there was a serious mistake in Problem 1, therefore both thresholds are now decreased.*