Fine-Grained and Parameterized Complexity
Problem Set 11, for **May 20th**, 2021

EPFL

Problem 1. Give an $\mathcal{O}(n^{\lceil k/2 \rceil})$ time algorithm for k-SUM.

Problem 2. Give a $\widetilde{\mathcal{O}}(n + d)$ time algorithm for 3SUM, where d denotes the largest absolute value of an integer in the input. Hint: Use FFT.

Problem 3. In the X+Y problem we are given two sets X and Y, each containing n integers, and we need to determine whether the sumset $X + Y = \{a + b \mid a \in X, b \in Y\}$ contains n^2 distinct integers. Prove that if the X+Y problem can be solved in truly subquadratic time, then 3SUM can also be solved in truly subquadratic time.

Problem 4. In the Unbounded Subset Sum problem we are given n integers $s_1, \ldots, s_n$ and a target value t, and we need to decide if there exist nonnegative integers $x_1, \ldots, x_n$ such that $\sum_{i \in [n]} x_i s_i = t$. Give a $\widetilde{\mathcal{O}}(n + t)$ time algorithm for Unbounded Subset Sum.

*This problem set adds **1 point** to the threshold for grade 4.0, and **2 points** for 6.0*