Problem 1. Prove that the Negative Triangle problem admits a truly subcubic algorithm if and only if the Negative Triangle problem restricted to tripartite graphs admits a truly subcubic algorithm.

Problem 2. In the Metricity problem we are given an \( n \times n \) matrix \( A \) with positive integer entries, and we have to determine whether for all \( i, j, k \in [n] \) we have \( A[i][j] \leq A[i][k] + A[k][j] \). Prove that Metricity is subcubic equivalent to APSP.

Problem 3. Prove that BMM Hypothesis is equivalent to the following hypothesis: Given an \( n \)-node undirected graph, no combinatorial algorithm can detect if the graph contains a triangle in time \( O(n^{3-\varepsilon}) \), for any \( \varepsilon > 0 \).

Problem 4. The Hitting Set Hypothesis says that, given sets \( S_1, \ldots, S_n, T_1, \ldots, T_n \subseteq [d] \), no algorithm can determine in \( O(n^{2-\varepsilon} \text{poly}(d)) \) time whether there is a set \( S_i \) that intersects every set \( T_j \). Prove that Hitting Set Hypothesis implies OV Hypothesis.

Problem 5. Design an \( O(n^{2.99}) \) time randomized algorithm for APSP in directed unweighted graphs. Hint: Consider short and long paths separately. For short paths, iteratively multiply the adjacency matrix. For long paths, sample a subset of vertices that hits all such paths with high probability, and compute shortest paths to and from that subset.

Problem 6. Prove that if \((\text{min},+)\)-product can be solved in \( T(n) \) time, then APSP can be solved in \( O(T(n)) \) time.

Hint: [link](https://people.csail.mit.edu/virgi/6.s078/hw2.pdf)

This problem set adds 2 points to the threshold for grade 4.0, and 4 points for 6.0