Problem 1. Prove that assuming SETH there is no algorithm that given three sets $U, V, W \subseteq \{0,1\}^d$ such that $|U| = |V| = |W| = n$ decides if there exist $u \in U$, $v \in V$, $w \in W$ such that $\sum_{i=1}^{d} u_i v_i w_i = 0$ in time $O(n^{3-\varepsilon}\text{poly}(d))$, for any $\varepsilon > 0$.

Problem 2. Prove that for every constant $\alpha > 0$ the Orthogonal Vectors Hypothesis is equivalent to the following hypothesis: There is no algorithm that given two sets $U, V \subseteq \{0,1\}^d$ such that $|U| = n$, $|V| = n^\alpha$ decides if there exist $u \in U$, $v \in V$ such that $u \cdot v = 0$ in time $O(n^{1+\alpha-\varepsilon}\text{poly}(d))$, for any $\varepsilon > 0$.

Problem 3. In the Longest Palindromic Subsequence problem we are given a string of length $n$, and we have to find its longest subsequence (of not necessarily consecutive elements) that is a palindrome (i.e. it reads the same backwards and forwards). Determine if the Longest Palindromic Subsequence problem can be solved in truly subquadratic time (i.e. in time $O(n^{2-\varepsilon})$ for some $\varepsilon > 0$)

Problem 4. Give an $O(n^2d^{0.373})$ time algorithm for Orthogonal Vectors restricted to instances with $d \leq n$.

Problem 5. Give an $O(n^{2.01})$ time algorithm for Orthogonal Vectors restricted to instances with $d \leq n^{0.1}$.

Problem 6. In the All-Pairs Hamming Distance problem we are given a set of $n$ strings, each of length $n$, and our goal is to compute for each pairs of strings $(u, v)$ the Hamming distance between them, i.e. $\sum_{i=1}^{n} 1[u[i] = v[i]]$. Give an $O(n^{2.99})$ time algorithm for that problem.

This problem set adds 1 point to the threshold for grade 4.0, and 4 points for 6.0