Problem 1. Prove that assuming SETH there is no algorithm that given three sets $U, V, W \subseteq \{0,1\}^d$ such that $|U| = |V| = |W| = n$ decides if there exist $u \in U$, $v \in V$, $w \in W$ such that $\sum_{i=1}^{d} u_i v_i w_i = 0$ in time $\mathcal{O}(n^{3-\varepsilon}\text{poly}(d))$, for any $\varepsilon > 0$.

Problem 2. Prove that for every constant $\alpha > 0$ the Orthogonal Vectors Hypothesis is equivalent to the following hypothesis: There is no algorithm that given two sets $U, V \subseteq \{0,1\}^d$ such that $|U| = n$, $|V| = n^\alpha$ decides if there exist $u \in U$, $v \in V$ such that $u \cdot v = 0$ in time $\mathcal{O}(n^{1+\alpha-\varepsilon}\text{poly}(d))$, for any $\varepsilon > 0$.

Problem 3. In the Longest Palindromic Subsequence problem we are given a string of length n, and we have to find its longest subsequence (of not necessarily consecutive elements) that is a palindrome (i.e. it reads the same backwards and forwards). Determine if the Longest Palindromic Subsequence problem can be solved in truly subquadratic time (i.e. in time $\mathcal{O}(n^{2-\varepsilon})$ for some $\varepsilon > 0$)

Problem 4. Give an $\mathcal{O}(n^2 d^{0.373})$ time algorithm for Orthogonal Vectors restricted to instances with $d \leqslant n$.

Problem 5. Give an $\mathcal{O}(n^{2.01})$ time algorithm for Orthogonal Vectors restricted to instances with $d \leqslant n^{0.1}$.

Problem 6. In the All-Pairs Hamming Distance problem we are given a set of n strings, each of length n, and our goal is to compute for each pairs of strings (u, v) the Hamming distance between them, i.e. $\sum_{i=1}^{n} \mathbb{1}\left[u[i] = v[i]\right]$. Give an $\mathcal{O}(n^{2.99})$ time algorithm for that problem.

*This problem set adds **1 point** to the threshold for grade 4.0, and **4 points** for 6.0*