Fine-Grained and Parameterized Complexity

Problem Set 7, for **April 15th**, 2021

Problem 1. Prove that $s_2 = 0$. Hint: show a subexponential (ideally polynomial) time algorithm for 2-SAT, and prove that its existence implies the equality.

Problem 2. Prove that SETH implies ETH. Hint: use Sparsification Lemma.

Hypothesis (Low-Dimensional Orthogonal Vectors Hypothesis (LDOVH))**.** For every $\varepsilon > 0$ there exists $c > 0$ such that the Orthogonal Vectors problem restricted to $d \leqslant c \cdot \log n$ cannot be solved in time $\mathcal{O}(n^{2-\varepsilon})$

Problem 3. Prove that SETH implies LDOVH, and that LDOVH implies OV Hypothesis.

Problem 4. In the Subset Sum problem we are given a set of n integers X and an integer target t, and we have to decide if there is a subset $A \subseteq X$ that sums up to t, i.e. $t = \Sigma_{a \in A} a$. Prove that there is no $2^{o(n)}$ time algorithm for Subset Sum unless ETH fails.

Problem 5. Given two sets of d-dimensional non-negative real-valued vectors $U, V \subseteq \mathbb{R}^d_{\geqslant 0}$, both of the same size $|U| = |V| = n$, the Maximum Inner Product problem asks to compute

$$\max\{u \cdot v \mid u \in U, v \in V\},$$

where "$\cdot$" denotes the inner product, i.e. $u \cdot v := \sum_{i=1}^{d} (u_i \cdot v_i)$. Prove that there is no $\mathcal{O}(n^{2-\varepsilon} \mathrm{poly}(d))$ time algorithm for Maximum Inner Product, for any $\varepsilon > 0$, unless OV Hypothesis fails.

Recall two common ways to define the Orthogonal Vectors problem:

Definition (Monochromatic Orthogonal Vectors)**.** Given a set of d-dimensional 0-1 vectors $U \subseteq \{0,1\}^d$, of size $|U| = n$, decide if there exists a pair of vectors $u, v \in U$ such that $u \cdot v = 0$.

Definition (Bichromatic Orthogonal Vectors)**.** Given two sets of d-dimensional 0-1 vectors $U, V \subseteq \{0,1\}^d$, both of the same size $|U| = |V| = n$, decide if there exists a pair of vectors $u \in U$ and $v \in V$ such that $u \cdot v = 0$.

Problem 6. Prove that Monochromatic Orthogonal Vectors and Bichromatic Orthogonal Vectors are *subquadratic equivalent*, i.e. if there exists an $\mathcal{O}(n^{2-\varepsilon} \mathrm{poly}(d))$ time algorithm for either one of them, for $\varepsilon > 0$, then there also exists an $\mathcal{O}(n^{2-\varepsilon'} \mathrm{poly}(d))$ time algorithm for the other, for $\varepsilon' > 0$.

This problem set adds ***1 point*** *to the threshold for grade 4.0, and* ***3 points*** *for 6.0*