Fine-Grained and Parameterized Complexity

Problem Set 6, for **April 1st**, 2021

EPFL

Problem 1. Given a graph G and an integer k, the Induced Matching problem asks to find an induced matching of size k, i.e., a set of k edges $(u_1, v_1), (u_2, v_2), \ldots, (u_k, v_k)$ such that the $2k$ endpoints are all distinct and there is no edge between $\{u_i, v_i\}$ and $\{u_j, v_j\}$ for any $i \neq j$. Prove that Induced Matching is W[1]-hard.

Problem 2. ~~In the Matching Strings problem we are given a set of n pairs of strings $(a_1, b_1), (a_2, b_2), \ldots, (a_n, b_n)$, over an alphabet of size k, and the goal is to find a nonempty sequence of indices $i_1, i_2, \ldots, i_\ell$ such that $a_{i_1} \odot a_{i_2} \odot \cdots \odot a_{i_\ell} = b_{i_1} \odot b_{i_2} \odot \cdots \odot b_{i_\ell}$, where $\odot$ denotes the string concatenation. Prove that Matching Strings parameterized by the alphabet size k is fixed parameter tractable.~~

Problem 3. Propose an algorithm that, given a CNF formula with n variables and m clauses, decides if the formula is satisfiable in time $\mathcal{O}(2^m \cdot \mathrm{poly}(n, m))$. Hint: solve the more general problem of *counting* the number of satisfying assignments, and to this end use the inclusion-exclusion principle.

Problem 4. In the Orthogonal Vectors problem we are given an n-element set of 0-1 vectors $A \subseteq \{0, 1\}^d$ and we need to decide if there exist two vectors $u, v \in A$ that are orthogonal, i.e., $\sum_{i=1}^{d} u_i v_i = 0$ (note that we compute the sum in $\mathbb{R}$, and not in $\mathbb{Z}_2$). Prove that for dimension $d = c \log n$ the Orthogonal Vectors be solved in $\mathcal{O}(n^{c+1})$ time.

Problem 5. The diameter of a graph is the largest distance (length of a shortest path) between any two vertices in this graph, or infinity if the graph is not (strongly) connected. Propose an algorithm that, given a directed unweighted graph $G = (V, E)$, finds, in time $\mathcal{O}(|V| + |E|)$, a 2-approximation of the diameter of G, i.e., an integer d such that the diameter is in $[d, 2d]$.

*This problem set adds ~~1 point~~ **0 points** to the threshold for grade 4.0, and ~~2 points~~ **1 point** for 6.0*

Problem 2 was an April fool – the Matching Strings problem is actually a famous undecidable problem, known under the name Post Correspondence Problem, hence no algorithm, not to mention an FPT one, can solve it.