Problem 1. Propose an $\mathcal{O}^*(4^w)$ time algorithm for the Dominating Set problem in graphs with a given tree decomposition of width at most w.

Problem 2. Propose an algorithm that finds the chromatic number of a graph with a given tree decomposition of width at most w running in time $w^{\mathcal{O}(w)} \cdot \mathrm{poly}(n)$.

Problem 3. Propose an algorithm that solves the Feedback Vertex Set problem in undirected graphs with a given tree decomposition of width at most w running in time $w^{\mathcal{O}(w)} \cdot \mathrm{poly}(n)$.

Problem 4. Propose a $2^{\mathcal{O}(\sqrt{k}\log k)} \cdot \mathrm{poly}(n)$ time algorithm for the Feedback Vertex Set problem in planar graphs parameterized by the solution size.

Problem 5. Propose an $\mathcal{O}^*(1.3803^k)$ time algorithm for Vertex Cover parameterized by the solution size. Hint: use (without a proof) the fact that every n-vertex graph of maximum degree 3 has treewidth at most $n/6+o(n)$ and the corresponding decomposition can be constructed in polynomial time.

Definition. A *monadic second order* formula is a formula that can use quantifiers over vertices, edges, and also over unary (hence monadic) relations (hence second order) over vertices or edges. For example, the following formula, saying that graph $G = (V, E)$ is connected, is a monadic second order formula.

$$\forall_{X\subseteq V} \left(\left(\exists_{u\in V}\ u \in X \wedge \exists_{u\in V}\ u \notin X\right) \implies \exists_{u\in V}\exists_{v\in V}\left((u,v) \in E \wedge u \in X \wedge v \notin X\right)\right)$$

Theorem 1 (Courcelle)**.** *Let ϕ be a monadic second order formula. There exists an algorithm that, given an n-vertex graph and its tree decomposition of width w, decides if ϕ is satisfied in that graph, in time $f(|\phi|, w) \cdot n$, for a computable function f.*

Problem 6. Use Courcelle theorem to prove that 3-Coloring is FPT parameterized by treewidth.

Remark. There exists also an optimization variant of Courcelle theorem. It says that one can optimize, in FPT time, over any affine function of sizes of the relations quantified existentially at the beginning of the formula.

Problem 7. Use Courcelle theorem to prove that Vertex Cover is FPT parameterized by treewidth.

*This problem set adds **1 point** to the threshold for grade 4.0, and **5 points** for 6.0*