1 Lower bounds: W[1]- and W[2]-hardness

The simplest way to prove that a problem is likely not FPT is to show that it is W[1]-hard. W[t] is the class of problems expresible as satisfiability of circuits of bounded depth and *weft* at most t. Weft of a circuit is the maximum number of gates of degree greater than two on any path from an input node to the output node. It is believed that

$$\mathrm{FPT} \subsetneq \mathrm{W}[1] \subsetneq \mathrm{W}[2] \subsetneq \cdots \subsetneq \mathrm{W}[t] \subsetneq \mathrm{W}[t+1] \subsetneq \cdots$$

though it is not implied by, say, P$\neq$NP.

The canonical W[1]-complete problem is (Max) Clique (parameterized by the solution size). Hence, providing a paramterized reduction from Clique to a given problem proves that this problem is likely not FPT.

The canonical W[2]-complete problem is (Min) Dominating Set (again, parameterized by the solution size). Providing a paramterized reduction from Dominating Set proves that the problem is even less likely to be FPT.

Exercise 1. Given a graph and an integer k, the Longest Induced Path problem asks to find in the graph an induced subgraph that is a path on k vertices. Show that Longest Induced Path is W[1]-hard.

2 Lower bounds: ETH and SETH

Please read Chapter 2 by Karl Bringmann and Marvin Künnemann, available at:
www.mpi-inf.mpg.de/fileadmin/inf/d1/teaching/summer19/finegrained/lec2.pdf