Problem 1. Given a set of n points in the plan and an integer k, the Point Line Cover problem asks to find k lines on the plane that contain all the input points. Show a polynomial kernel for this problem.

Problem 2. A graph is a *cluster graph* if each of its connected components is a clique. Given a graph G and an integer k, the Cluster Editing problem asks to determine if there exist k edge *modifications* (i.e. insertions or removals) that transform G into a cluster graph. Show an $\mathcal{O}^*(3^k)$ time algorithm for this problem. Hint: What does every non-cluster graph contain?

Problem 3. Given a graph G, its vertex cover W, and an integer k, the Disjoint Vertex Cover problem asks if there exists a vertex cover of G disjoint from W of size at most k. Show a polynomial time algorithm for this problem.

Problem 4. Given a graph G and its vertex cover Z of size k, the Vertex Cover Compression problem asks if there exists a vertex cover of G of size $k-1$. Show an $\mathcal{O}^*(2^k)$ time algorithm for this problem. Hint: Call a Disjoint Vertex Cover algorithm 2^k times.

Problem 5. Solve Vertex Cover in $\mathcal{O}^*(2^k)$ time by calling a $\mathcal{O}^*(2^k)$ time algorithm for Vertex Cover Compression $n-k$ times. Hint: Start with a k-vertex subgraph of the input graph.

*This problem set adds **0 points** to the threshold for grade 4.0, and **1 point** for 6.0*

During the session we solved only Problems 1 and 2.
Problems 3, 4, and 5 will appear again in the next problem set.