Deterministic 3SUM hardness

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The 3SUM problem – given n numbers decide if any three of them sum up to zero – is one of the core problems in fine-grained complexity. It can be solved in $O(n^2)$ time, and there is a hypothesis that no $O(n^{2-\varepsilon})$-time algorithm can solve it.

Following Pătraşcu's seminal paper [Pat10] 3SUM was reduced to many different combinatorial problems, proving that they are hard under the aforementioned hypothesis, see, e.g., [AW14, KPP16]. All these reductions start with a single trick, which is based on the so called almost linear hashing, and thus they are inherently randomized. Recently Chan and He [CH20] gave a simple and deterministic reduction from 3SUM to Convolution-3SUM.

The goal of this project is to look for more of the previously known reductions from 3SUM which can be derandomized with similar techniques.

References

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