Question 1:

Prove that for every graph $G$ the product of its chromatic number and its independence number is at least as large as its number of vertices.

*For this question you are not allowed to use any results from the lecture.*

Solution:
Question 2:
Let $G = (V, E)$ be a graph and $v \in V$ a vertex. Show that if $v$ has at least $k$ neighbors in $G$ and $G' = G - v$ is $k$-connected, then $G$ is $k$-connected.

Solution:
Question 3:
Let $G$ be a road network on which there are $m$ factories $f_1, \ldots, f_m$ and $n$ villages $v_1, \ldots, v_n$. Goods need to be shipped from factories to villages on this road network. Factory $f_i$ produces goods at rate $p_i$, village $v_i$ demands goods at rate $d_i$ and the roads all have capacity $c$.
Show that $G$ can be transformed into a network $G'$ such that all demands are satisfied if and only if the value of the max flow on $G'$ is equal to the sum of the villages' demands.

Solution: