Graph theory - problem set 12

December 12, 2019

1. Calculate the eigenvalues and eigenvectors of the adjacency matrix of C_4.
2. (a) Let G be a graph, and let k be a positive integer. Prove that for every $x, y \in V(G)$, $A_G^k(x, y)$ is equal to the number of walks of length k in G with endpoints x and y.

   (b) Let G be a graph on n vertices and let $\lambda_1, ..., \lambda_n$ be all the eigenvalues of A_G. Show that

$$\sum_{i=1}^{n} \lambda_i^2 = 2|E(G)|.$$

3. Let G be a graph that is $\mathrm{sgr}(n, d, \lambda, \mu)$. Calculate n as a function of d, λ and μ.
4. Let G be a d-regular graph. Prove that if λ is an eigenvalue of A_G, then $|\lambda| \leq d$.
5. Let G be a bipartite graph. Prove that if λ is an eigenvalue of A_G, then $-\lambda$ is also an eigenvalue.
6. Let G be a graph and let p be the number of positive eigenvalues of A_G (with multiplicity), and let n be the number of negative eigenvalues of A_G (with multiplicity). Prove that the edge set of G cannot be partitioned into fewer than $\max(p, n)$ complete bipartite graphs.
7. Let G be a graph that is $\mathrm{sgr}(n, d, \lambda, \mu)$. Calculate the eigenvalues of A_G as a function of n, d, λ, μ.