Graph theory - problem set 2

September 26, 2019

Exercises

1. Prove the triangle-inequality in graphs: for any three vertices u, v, w in a graph G,
$$d(u, v) + d(v, w) \geq d(u, w).$$
2. Show that a graph is connected if and only if it contains a spanning tree.
3. A graph that does not contain any cycles is called a forest.
Prove that a forest on n vertices with c connected components has exactly $n - c$ edges.
4. Let T be a tree and e be an edge of T. Prove that $T - e$ is not connected.
5. Let T be a tree and let u and v be two non-adjacent vertices of T. Prove that $T + uv$ contains a unique cycle.
6. Let W be a closed walk that uses the edge e exactly once. Prove that W contains a cycle through e.
7. Prove that every connected graph on $n \geq 2$ vertices has a vertex that can be removed without disconnecting the remaining graph.
8. Show that every tree T has at least $\Delta(T)$ leaves. (Where $\Delta(T)$ is the maximum degree of T.)
9. Let T be a tree on t vertices and suppose G is a graph with $\delta(G) \geq t - 1$. Show that $T \subseteq G$, i.e., G has a subgraph isomorphic to T.
10. Let G be a graph on n vertices. Prove that if G has at least $2n - 1$ edges, then it contains an even cycle.