Definition : For a directed graph $G = (V, A)$ with weight function $\ell : A \to \mathbb{R}$, we define a potential function to be a function $d : V \to \mathbb{R}$. The reduced weight $\ell_d : A \to \mathbb{R}$ corresponding to $d$ is given by:

$$\ell_d(u, v) = \ell(u, v) + d(u) - d(v)$$

for all $(u, v) \in A$.

Let $G = (V, A)$ be a directed graph with weight function $\ell : A \to \mathbb{R}$ and suppose that $G$ has no negative cycles.

1. Let $P$ be a shortest path between $s$ and $t$ with respect to $\ell$. Show that for each potential function, $P$ is also a shortest path between $s$ and $t$ in respect to the reduced weights.

2. Show that there exists a potential function $d^+$ such that all reduced weights are $\geq 0$. 

---

1. You are allowed to submit your solutions in groups of at most three students.