Discrete optimization | Thursday, Mai 23, 2019
Prof. F. Eisenbrand | EPFL

The problem can be submitted until Mai 31, 12 :00 noon, into the box in front of MA C1 563 (Attention : There is no exercise session in this week).

Student(s) [1] :

Question 1 : *The question is worth* 5 *points.*

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 *Reserved for the corrector*

Definition : For a directed graph $G = (V, A)$ with weight function $\ell : A \to \mathbb{R}$, we define a *potential function* to be a function $d : V \to \mathbb{R}$. The *reduced weight* $\ell_d : A \to \mathbb{R}$ corresponding to d is given by :

$$\ell_d(u, v) = \ell(u, v) + d(u) - d(v)$$

for all $(u, v) \in A$.

Let $G = (V, A)$ be a directed graph with weight function $\ell : A \to \mathbb{R}$ and suppose that G has no negative cycles.

1. Let P be a shortest path between s and t with respect to ℓ. Show that for each potential function, P is also a shortest path between s and t in respect to the reduced weights.
2. Show that there exists a potential function d^+ such that all reduced weights are ≥ 0.

1. You are allowed to submit your solutions in groups of at most three students.