Discrete optimization | Thursday, Mai 10, 2019
Prof. F. Eisenbrand | EPFL

The problem can be submitted until Mai 17, 12 :00 noon, into the box in front of MA C1 563 or during the exercise session.

Student(s) [1] :

Question 1 : *The question is worth* 5 *points.*

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 *Reserved for the corrector*

For a polyhedron P with vertices V and edges E, we can consider the graph $G = (V, E)$. The combinatorial diameter of this graph, $diam(P)$, is the minimum number of edges needed in order to reach any vertex by a path from any other vertex. Obtaining a good bound on $diam(P)$ is a very difficult and unsolved problem, see for instance the Hirsch conjecture. But for the special case where P is bounded and the vertices of P are in $\{0, 1\}^n$, Naddef has shown (and you will too...) that $diam(P) \leq n$. It might be useful to follow these steps :

1. Show the theorem for $n = 1$. (It might be useful to think about why it also works for $n = 2$).
2. Show that $P_1 := \{x \in \mathbb{R}^n \mid e_1^T x = 1\} \cap P$ corresponds to a polyhedron in dimension $n - 1$ with vertices in $\{0, 1\}^{n-1}$.
3. Show that set of vertices and edges of P_1, V_1 and E_1, is a subset of the vertices of P, i.e. $V_1 \subseteq V$ and $E_1 \subseteq E$.
4. If we are at vertex $v \in P$ with $v_1 = 0$ and there exists another vertex $w \in P$ with $w_1 = 1$, show that there must be a neighbour $\bar{v}$ of v (i.e. there is an edge between v and $\bar{v}$) such that $\bar{v}_1 = 1$.
5. Conclude.

1. You are allowed to submit your solutions in groups of at most three students.