École Polytechnique Fédérale de Lausanne
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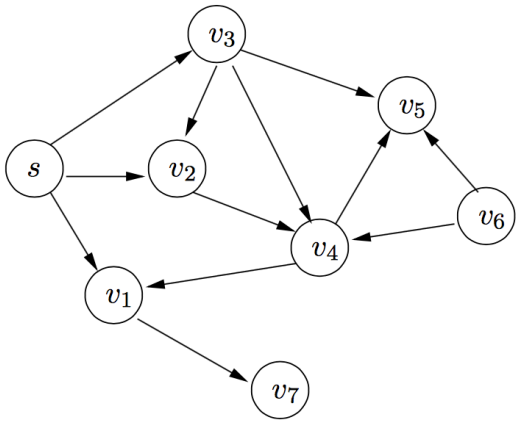
Discrete Optimization (Spring 2019)

Assignment 11

Problem 1

In an unweighted graph (i.e., where all edges are of the unit weight) the shortest path from s to all other vertices can be computed by using the breadth-first search (BFS) algorithm. Apply it to the graph bellow.



Consider each iteration of the BFS (i.e. each time when a new vertex is taken from the head of the queue). Note which vertex has been currently processed, the distance labels and the snapshot of the queue at the end of the iteration.

Problem 2

There are n types of animals, and you want to assign them to two stables. Unfortunately, some animals would eat other animals when left unattended. Therefore you need to assign the animals carefully. There are m relations of the form "u eats v", where u and v are animals.
Find an $O(n+m)$ algorithm that decides whether there is an assignment of animals to the two stables such that no animal eats another one of the same stable, and outputs a feasible assignment.

Hint: Observe that the problem is equivalent to checking if the underlying (undirected) graph $G = (V, E)$ is bipartite.

Problem 3

Consider a directed graph $D = (V, A)$ with n vertices and m arcs. A toplogical sort of the vertices is a total ordering on V such that there is no arc $(u, v) \in D$ such that $u > v$, i.e, such that u is placed after v in the ordering.

Formulate an $O(m+n)$ algorithm that finds a topological sort of the vertices or decides that there is a directed cycle in G.

Problem 4

Let $D = (V, A)$ be a directed *acyclic* graph, i.e., there exists no directed cycle in D, and let $w : A \to \mathbb{R}$ be arc weights. Assume that you are given a topological sort of the vertices. Show how the above ordering can be used to compute single-source shortest paths, with respect to w, in $O(m)$ arithmetic operations.

Problem 5

Let $T = (V, E)$ be an undirected tree. Find an algorithm which in time $O(|V|)$ finds the longest distance between any two vertices in V, i.e., $\max_{u,v \in V} d(u, v)$.