École Polytechnique Fédérale de Lausanne
Prof. F. Eisenbrand

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Discrete Optimization (Spring 2019)

Assignment 10

Problem 1

Prove Hall's theorem: Let $G = (A \cup B, E)$ be a bipartite graph, and for each $S \subseteq A$, let

$$N(S) = \{v \in B : \ \exists u \in S \text{ such that } \{u, v\} \in E\}.$$

Then, G has a matching of size $|A|$ if and only if $|N(S)| \geq |S|$ for all $S \subseteq A$.

Problem 2

Show that the node-edge incidence matrix A of some graph G is totally uninmodular, if and only if G is bipartite.

Problem 3

Consider a graph $G = (V, E)$. A matching $M \subseteq E$ is said to be *maximal* if there is no edge $e \in E \setminus M$ such that $M \cup e$ is a matching. Denote with M^* a maximum cardinality matching in G.

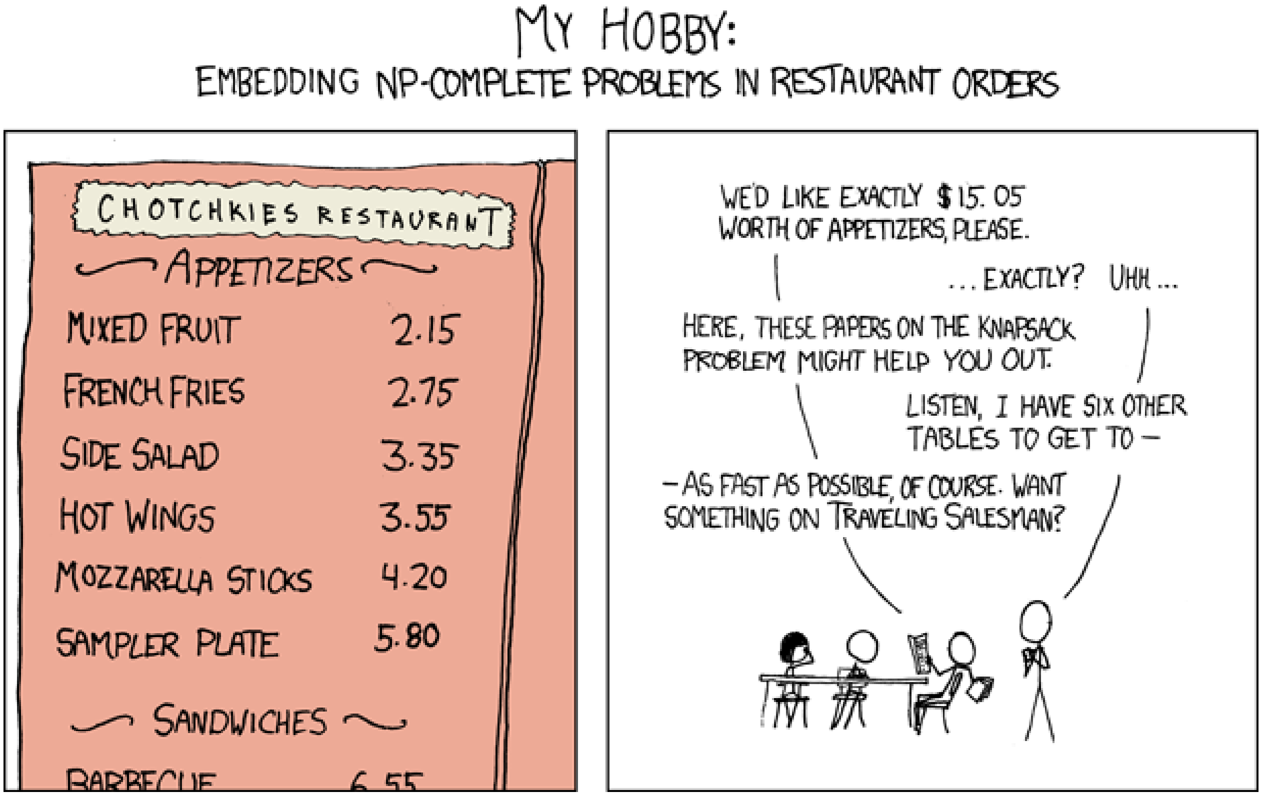
a) Show that $|M| \geq \frac{|M^*|}{2}$ for any maximal matching M in G.

b) Provide a graph containing a maximal matching M with $|M| = \frac{|M^*|}{2}$.

Problem 4

Let $\max\{c^T x \colon Ax \leq b, \, x \geq 0, x \in \mathbb{Z}^n\}$ be an integer program that has feasible integer solutions. Prove the following: If the LP-relaxation is unbounded, then so is the integer program.
Give an example of an infeasible integer program whose LP relaxation is unbounded.

Problem 5



For more exercises or comics, please go to xkcd.com