The problem can be submitted until May 3, 12:00 noon, either at the exercise session or into the box in front of MA C1 563.

Student(s) 1 :

Question 1: The question is worth 5 points.

$$\square \ 0 \ \square \ 1 \ \square \ 2 \ \square \ 3 \ \square \ 4 \ \square \ 5$$
 Reserved for the corrector

Given $c \in \mathbb{R}^n_+$, $a \in \mathbb{R}^n_+$ and $\gamma \in \mathbb{R}_+$, design an algorithm which, in $O(n \log n)$ operations, computes the optimal solution x^* to the following linear program :

$$\begin{aligned} \max \quad c^T x \\ \text{s.t.} \quad a^T x &\leq \gamma, \\ 0 &\leq x_i \leq 1, \quad \forall i \in [n]. \end{aligned}$$

You may assume that a set of n real numbers can be sorted in time $O(n \log n)$ and that each arithmetic operation takes constant time. It is important to prove that the solution returned by the designed algorithm is optimal.

^{1.} You are allowed to submit your solutions in groups of at most three students.