Discrete optimization | Thursday, April 12, 2019
Prof. F. Eisenbrand | EPFL

The problem can be submitted until April 19, 12 :00 noon, into the box in front of MA C1 563, there will be no exercise session on April 19.

Student(s) [1] :

Question 1 : *The question is worth* 5 *points.*

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 *Reserved for the corrector*

We define $\mathcal{P} = \{P \in \{0,1\}^{n \times n} \mid P \text{ invertible and the sum of all its entries } \leq n\}$.
Furthermore, $Q \in \mathbb{R}^{n \times n}$ is *funny*, if the entries of any row and of any column are positive and sum up to 1. For example, the following is a funny matrix :

$$\begin{bmatrix} 1/5 & 1/5 & 3/5 \\ 2/5 & 3/5 & 0 \\ 2/5 & 1/5 & 2/5 \end{bmatrix}.$$

Show that for every funny matrix Q we have :

$$Q \in conv(\mathcal{P})$$

(this means that Q can be written as a convex combination of elements in $\mathcal{P}$)
Hint : The set $\mathcal{P}$ has an easy description. You may also want to express the set of all funny matrices as a polyhedron. How do its extreme points look like ?

1. You are allowed to submit your solutions in groups of at most three students.