The problem can be submitted until April 12, 12 :00 noon, either at the exercise session or into the box in front of MA C1 563.

Student(s) [1] :

Question 1 : *The question is worth 5 points.*

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 *Reserved for the corrector*

Given the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 6 & 3 & 7 \\ 2 & 4 & 3 \end{pmatrix}$, find optimal mixed row and column strategies and the value of the zero-sum game defined by A.

Sol.:

Observe that the first row can be removed since the row player will always prefer the other two. After doing this, the column player will clearly prefer the first column to the third. Thus we obtain an equivalent problem defined with the matrix $\bar{A} = \begin{pmatrix} 6 & 3 \\ 2 & 4 \end{pmatrix}$.

Here, we can solve the corresponding LP as in Problem 1 of Assignment 7. Alternatively, since the matrix is 2×2 *we can directly derive optimal strategies for both players. Let* $0 \leq p \leq 1$ *and* $0 \leq q \leq 1$*, consider the following quantity :*

$$\begin{pmatrix} p & 1-p \end{pmatrix} \begin{pmatrix} 6 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} q \\ 1-q \end{pmatrix} = 4 - p - 2q + 5pq = 18/5 + 5(p - 2/5)(q - 1/5).$$

If $p \neq 2/5$ *then the column player can set* q *so that the expression* $5(p - 2/5)(q - 1/5)$ *is negative. Analogously, in the case of* $q \neq 1/5$, p *can make the same expression positive. Thus, optimal strategies are achieved for* $p = 2/5$ *and* $q = 1/5$*. This translates into optimal row strategy* $x^* = (0, 2/5, 3/5)^T$ *and column strategy* $y^* = (1/5, 4/5, 0)^T$*, and the value of the game* $18/5$.

1. You are allowed to submit your solutions in groups of at most three students.