École Polytechnique Fédérale de Lausanne
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Discrete Optimization (Spring 2019)

Assignment 8

Problem 1

Show that:

$$a^{\log(b)} = b^{\log(a)}$$

for any $a, b > 0$.

Problem 2

For each recurrence relation, give the correct asymptotic growth for the following expressions (assuming $T(0) = 0$, $T(1) = 1$ and $T(\cdot) = T(\lfloor \cdot \rfloor)$):

- $T(n) = T(n-1) + 1$
- $T(n) = 16T(n/4) + n^2$
- $T(n) = 7T(n/2) + 5n^2$
- $T(n) = T(n^{1/2}) + \log(n)$
- $T(n) = T(n - \sqrt{n}) + T(5)$

Problem 3

Complete the algorithm below such that it adds two natural numbers in binary representation $a_0, \ldots, a_{l-1},\ b_0, \ldots, b_{l-1}$. What is the asymptotic running time (number of basic operations) of your algorithm? Can there be an asymptotically faster algorithm?

Input: Two natural numbers a and b in their binary representation $a_0, \ldots, a_{l-1},\ b_0, \ldots, b_{l-1}$.
Output: The binary representation $c_0, \ldots, c_l$ of $a + b$

carry := 0
for $i = 0, \ldots, l-1$
  $c_i = \text{carry} + a_i + b_i \pmod 2$
  carry :=
c_l :=
return $c_0, \ldots, c_l$

Problem 4

Show that there are n-bit numbers $a, b \in \mathbb{N}$ such that the Euclidean algorithm on input a and b performs $\Omega(n)$ arithmetic operations.
Hint: Fibonacci numbers

Problem 5

Let $A \in \mathbb{R}^{m \times n}$. $A_{j \longleftrightarrow i}$ is the matrix A where we switch rows i and j. Similarly, $A_{i \to i + cj}$ is the matrix A where we add c times row j to row i and leave the rest unchanged.

1. Find a matrix S_{ij} such that $S_{ij} A = A_{j \longleftrightarrow i}$ and a matrix E_{ij}^c such that $E_{ij}^c A = A_{i \to i + cj}$
2. Given i, j, k, l, c, find i', j', k', l', c' such that:

$$S_{kl} E_{ij}^c = E_{i'j'}^{c'} S_{k'l'}$$