The problem can be submitted until March 22, 12:00 noon, either at the exercise session or into the box in front of MA C1 563.

Student(s) 1 :

Question 1: The question is worth 5 points.

$$\square \ 0 \ \square \ 1 \ \square \ 2 \ \square \ 3 \ \square \ 4 \ \square \ 5$$
 Reserved for the corrector

Let $P = \{x \in \mathbb{R}^n \mid Ax \leq b\}$ be a polyhedron. Two extreme points $x_1, x_2 \ (x_1 \neq x_2)$ of P are said to be *adjacent* if there exists a valid inequality $d^Tx \leq \beta$ of P such that

$$P \cap \{x \in \mathbb{R}^n \mid d^T x = \beta\} = \{\lambda x_1 + (1 - \lambda)x_2 \mid \lambda \in [0, 1]\}$$

Show the following statement: Two different vertices v_1 and v_2 are adjacent, if and only if there exist two bases $B_1, B_2 \subset \{1, \ldots, m\}$ such that $|B_1 \cap B_2| = n - 1$ and $v_1 = A_{B_1}^{-1} b_{B_1}$ as well as $v_2 = A_{B_2}^{-1} b_{B_2}$ holds.

^{1.} You are allowed to submit your solutions in groups of at most three students.