The problem can be submitted until March 15, 12 :00 noon, either at the exercise session or into the box in front of MA C1 563.

Student(s) [1] :

Question 1 : *The question is worth 5 points.*

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5	*Reserved for the corrector*

Consider a linear program $\max\{c^T x\colon x \in \mathbb{R}^n, Ax \leq b\}$ with $A \in \mathbb{R}^{m\times n}$ full column rank and a feasible basis $B \subseteq \{1,\dots,m\}$. (Recall that a basis is feasible if $x^*_B = A_B^{-1} b_B$ is a feasible solution, in fact a vertex.) The aim is to show that, if B is not an optimal basis, x^*_B is not an optimal solution under a certain condition.

(i) Show that there exists a unique $\lambda \in \mathbb{R}^m$ such that

$$\lambda^T A = c^T \text{ and } \lambda_j = 0 \text{ for each } j \notin B.$$

(ii) Let $i \in B$. Show that there exists a unique $d_i \in \mathbb{R}^n$, $d_i \neq 0$ such that

$$a_j^T d_i \begin{cases} 0 & \text{for } j \in B \setminus \{i\} \\ -1 & \text{if } j = i. \end{cases}$$

Show that $\lambda_i < 0$ implies $c^T d_i > 0$.

(iii) Conclude that, if the inequalities that are tight at x^*_B are those indexed by B only, then x^*_B is not optimal.

1. You are allowed to submit your solutions in groups of at most three students.