Discrete optimization — Thursday, March 1, 2019
Prof. F. Eisenbrand — EPFL

The problem can be submitted until March 8, 12 :00 noon, either at the exercise session or into the box in front of MA C1 563.

Student(s) [1] :

Question 1 : *The question is worth 5 points.*

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5	*Reserved for the corrector*

Assume validity of the following form of the Farkas' lemma

> Let $A \in \mathbb{R}^{m\times n}$ be a matrix and $b \in \mathbb{R}^m$ be a vector. The system $Ax = b$, $x \geq 0$, $x \in \mathbb{R}^n$ has a solution if and only if for all $\lambda \in \mathbb{R}^m$ with $\lambda^T A \geq 0$, one has $\lambda^T b \geq 0$.

Prove the following variant of Farkas' lemma (Theorem 3.11) :

> Let $A \in \mathbb{R}^{m\times n}$ be a matrix and $b \in \mathbb{R}^m$ be a vector. The system $Ax \leq b$, $x \in \mathbb{R}^n$ has a solution if and only if for all $\lambda \in \mathbb{R}^m_{\geq 0}$ with $\lambda^T A = 0$ one has $\lambda^T b \geq 0$.

Sol. : The system $Ax \leq b$ has a solution if and only if the system $\hat{A}\hat{x} = b$ has a positive solution $\hat{x} \geq 0$ where $\hat{A} = [A - A\, I_m]$ and $\hat{x} \in \mathbb{R}^{(2n+m)\times 1}$. Thus, by the Farkas lemma, this happens if and only if for all $\lambda \in \mathbb{R}^m$ such that $\lambda^T A \geq 0$, one has $\lambda^T b \geq 0$. Since $\lambda^T[A - A\, I_m] \geq 0$ is equivalent to $\lambda \geq 0$ and $\lambda^T A = 0$ (since $\lambda^T A, -\lambda^T A \geq 0$), the variant of Farkas' lemma is proven.

1. You are allowed to submit your solutions in groups of at most three students.