École Polytechnique Fédérale de Lausanne
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Discrete Optimization (Spring 2019)

Assignment 5

Problem 1

Consider a feasible LP $\max\{c^{\mathrm{T}}x \ : x \in \mathbb{R}^n, Ax \leq b\}$ with $\operatorname{rank}(A) = n$. Let B be an optimal basis and λ_B such that $\lambda_B^{\mathrm{T}} A_B = c^{\mathrm{T}}$. Prove or give a counter-example for the following statements:

i) If λ_B is strictly positive, then the optimal solution is unique.

ii) If the optimal solution is unique, then λ_B is strictly positive.

Problem 2

Prove that the truthfulness of the statement in Problem 1.ii) changes if we assume that the considered polyhedron is non-degenerate.

Problem 3

Suppose you are given an oracle algorithm, which for a given polyhedron

$$P = \{\tilde{x} \in \mathbb{R}^{\tilde{n}} \ : \ \tilde{A}\tilde{x} \leq \tilde{b}\}$$

gives you a feasible solution or asserts that there is none. Show that using a *single* call of this oracle one can obtain an *optimum* solution for the LP

$$\max\{c^T x \ : \ x \in \mathbb{R}^n; \ Ax \leq b\},$$

assuming that the LP is feasible and bounded.
Hint: Use duality theory!

Problem 4

Consider the following linear program:

$$\begin{aligned} \max \quad & x_1 + x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \leq 6 \\ & x_1 + 2x_2 \leq 8 \\ & 3x_1 + 4x_2 \leq 22 \\ & x_1 + 5x_2 \leq 23 \end{aligned}$$

Show that $(4/3, 10/3)$ is an optimal solution by using duality.

Problem 5

Implement Phase II of the Simplex algorithm, i.e., solve the LPs defined by A, b, c, given their initial feasible bases. Use the file "Simplex.py" which can be found on the course git server.